UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 7th FEBRUARY 2002

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

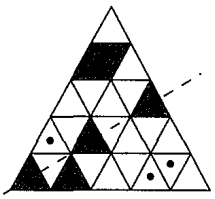
This solutions leaflet for the IMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **D** $3 = \frac{15}{5}$ and $4 = \frac{20}{5}$; 19 is the only one of the options between 15 and 20.

2. **D** $0.3 \times 7 = 2.1$; $0.5 \times 5 = 2.5$; $0.2 \times 11 = 2.2$; $0.09 \times 30 = 2.7$ and $0.026 \times 100 = 2.6$.

3. **C** When 17 is divided by 3, there is a remainder of 2. Thus there are either 2 chairs, 5 chairs, or more. Since 5 chairs have too many legs there must be 2 chairs and 3 stools.

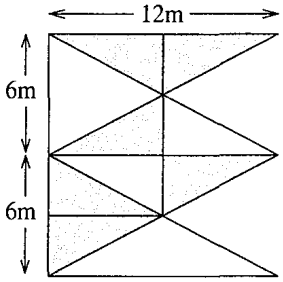
4. **B** Taking the altitude shown, symmetry can be obtained by shading 3 more triangles. Taking the vertical altitude would require 5 more to be shaded and the other sloping altitude would need 6 more.



5. **E** The cost for 9 apples would be $(24 + 23 + \ldots + 16) = 180$ pence.

6. **C** $6 \times 3 = 18$; $7 \times 5 = 35$; $8 \times 5 = 40$; $9 \times 4 = 36$; $10 \times 3 = 30$.

7. **B**



The diagram shows that the required area is made up of six congruent triangles. Thus the area, in m^2, is $6 \times (\frac{1}{2} \times 6 \times 3) = 6 \times 9$.

8. **D** $2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$. So the answer comes from $1024 - 100$.

9. **B** A has adjacent 3s. In C, the 3s are just one digit apart. In D, the 3s are just two digits apart. In E, the 4s are adjacent. But B satisfies all the requirements.

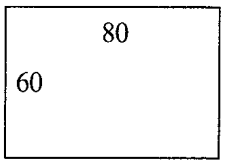
10. **C** In the family, there are 3 boys and 6 girls. So Tom has 2 brothers and 6 sisters.

11. **D** Let the first term be x. Then the sequence is: x, 4, $x + 4$, $x + 8$, $2x + 12$. So $2x + 12 = 22$ and hence $x = 5$.

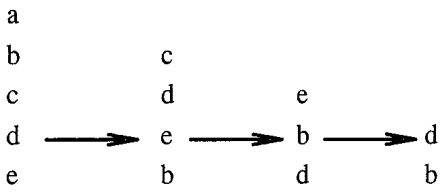
12. **A** Along the bottom, there is a pair of black counters which is a threat to the second player. If Black plays at D then White can play at A *but* if Black plays at A then White is helpless. If Black plays at E, White can play at C and vice versa. If Black plays at B, then White can play to the right of B, blocking that row.

13. **E** The guesses were 5040, 5060, 5110, 5120, 5150. The range of these guesses is just 110 so the two 'wild' estimates must both be low or both high. Since the difference between 5040 and 5060 is 20, 5040 is 90 out and 5060 is 70 out.

14. E By the Theorem of Pythagoras, the diagonal of the window is 100cm which exceeds the length or the breadth of all the sheets. So the first three pieces can go through the window either way and the 90 cm × 105 cm piece can also go through the window provided that a 90 cm edge goes first.

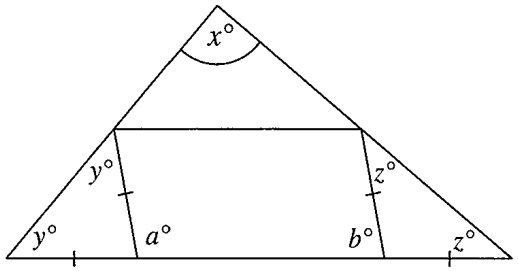


15. D



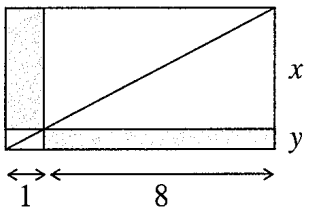
The sequence of cards I am given is a, c, e, d, b. This gives a = Ace; c = 2; e = 3; d = 4; b = 5. Hence the original order was Ace, 5, 2, 4, 3.

16. C The two angles marked $y°$ are equal because they are in an isosceles triangle. For the same reason, the angles $z°$ are equal. Since an exterior angle of a triangle is the sum of the two interior and opposite angles, it follows that $a = 2y$ and $b = 2z$. Now $a° + b° = 180°$ since they are the base angles of a parallelogram. So $2y + 2z = 180$ giving $y + z = 90$. But, from the angle sum of a triangle $x + y + z = 180$; hence $x = 90$.



17. B Let the distance to the bike shop be d miles. Then the time going is $\frac{1}{3}d$ hours and the time returning is $\frac{1}{12}d$ hours. The total travelling time is thus $\frac{1}{3}d + \frac{1}{12}d = \frac{5}{12}d$ hours and the total distance is $2d$ miles giving the average speed as $(2d) \div (\frac{5}{12}d) = 2 \times \frac{12}{5} = \frac{24}{5}$ miles per hour.

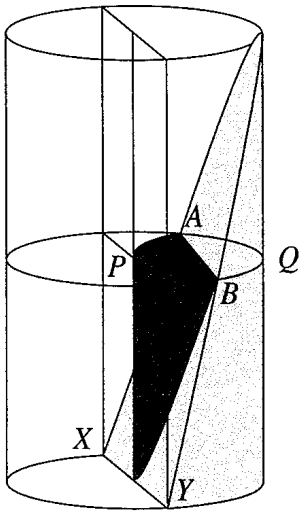
18. A



Let x and y be the distances shown. Then the shaded area is $8y + x$. But there are a number of similar triangles and from one pair $\frac{x}{8} = \frac{y}{1}$ i.e. $x = 8y$. So,

$$\frac{\text{shaded area}}{\text{total area}} = \frac{8y + x}{9(x + y)} = \frac{8y + 8y}{9 \times 9y} = \frac{16}{81}.$$

19. A Imagine the liquid became solidified. Rotate the can until the face which contains XY is horizontal.
Now take the horizontal cross-section half way up the can which meets the plane face of the 'liquid' in the line AB. Observing that AB bisects the radius PQ, rotate the upper section of the 'liquid' about AB through 180°. The new position is shaded darkly and fits into the bottom right quarter (with some space to spare).
[The actual fraction may be shown to be $2 / (3\pi)$, i.e. approximately 0.21.]



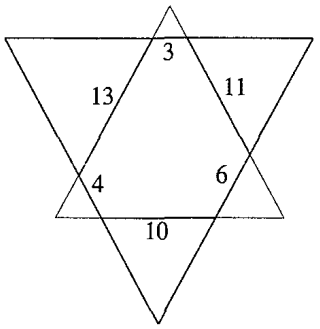
20. **A** The maximum score is 135. If a candidate omits one of the first 15 questions, or answers it wrongly, then effectively 5 marks are lost from a possible maximum score. Similarly, if one of questions 16 to 25 is omitted then effectively 6 marks are lost. If one of questions 16 to 20 is answered incorrectly then effectively 7 marks are lost and an incorrect answer to one of questions 21 to 25 effectively loses 8 marks. Thus scores of 130, 129, 128 and 127 are all possible but 126 is not possible since one omission or incorrect answer effectively loses a maximum of 8 marks, while two omissions or incorrect answers lead to an effective loss of at leas 10 marks.

21. **D**

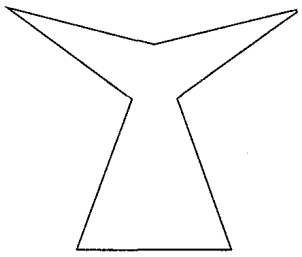
Year	2002	2003	2004	2005	2006	2007	2008
1st January	Tu	Wed	Thu	Sat	Sun	Mon	Tue
	2009	2010	2011	2012	2013	2014	
	Thu	Fri	Sat	Sun	Tue	Wed	

So two years in the list start on a Tuesday. But *each* date has to fall on the same day and since 2008 is a leap year, it must be excluded.

22. **C** By extending the short sides, one can construct another triangle. Since all the angles involved are 60°, the new small triangles and the new large triangle are all equilateral. The sides of the new, large triangle are
$13 + 3 + 11 = 11 + 6 + 10 = 10 + 4 + 13 = 27$.



23. **E** The description does not say that the heptagon has to be convex, i.e. all of its interior angles need not be less than 180°. Since $(2 \times 7 - 4) \times 90 = 900$ the interior angles of all heptagons total 900°. The creation of a heptagon with all the given conditions is possible as the diagram shows. Notice that four of the interior angles are acute and the other three are *reflex* angles.



24. **C** To satisfy the stated condition, the display will have the form $10{:}m_1m_2{:}s_1s_2$. The values of both m_1 and s_1 have to be chosen from 2, 3, 4, 5. So there are four ways of choosing m_1 and then three choices for s_1. Since four digits have been chosen, m_2 and s_2 are selected from the remaining six. Thus the total number of times is $4 \times 3 \times 6 \times 5 = 360$.

25. **B** $x = 1 - \frac{1}{111111} = 1 - \frac{6}{666666}$; $y = 1 - \frac{2}{222223} = 1 - \frac{6}{666669}$;
$z = 1 - \frac{3}{333334} = 1 - \frac{6}{666668}$. Now $\frac{6}{666669} < \frac{6}{666668} < \frac{6}{666666}$ and

so $x < z < y$.